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EARTHWORKS CALCULATION – ANALYSIS OF EXISTING FORMULAS AND SHOWCASING THE FUTURE METHOD FOR VOLUME CALCULATION

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Abstract

Calculation of quantities in the design of infrastructure structures has been calculated for many years by use of classical formula of the Arithmetic mean between two cross sections multiplied by the length between them. This equation has been synonymous with the accurate calculation of quantities in infrastructure projects. Other equations that have been used, but not so frequently, are the prismoid formula or mid sectional area method. This paper analyses the accuracy of several equations (formulas) for calculating quantities and compares the results with the volume of 3D modelled terrain on different types of sections, terrain category and type of infrastructure road. 3D modelled terrain is a 3D solid element derived by calculating volume surface. The volume surface is a difference between a base surface and comparison surface which means that Z-value (vertical value) of any given point is the difference between Z-values of the surfaces. In the infrastructure context, volume surface calculates the difference between new top surface or planum surface and existing ground which provide accurate calculation results. The purpose of this paper is also to examine using volume surface method as an appropriate substitute method for volume calculation.

Keywords: earthworks, calculations, formula, volume calculation

1 Introduction

This article will analyse the accuracy of 4 different equations for calculating volume of earthworks and subsequently compare it with the volume surface method. The idea for this scientific was formed when I was asked to track the earthworks based on topology produced by the drone. It was clear that the best way to calculate the earthworks was to compare the triangulated surface from the previous drone flight with the new topology and subtract the surfaces. With that method you could easily track the completed works. As I was working on this project, a question formed in my mind as to why the road engineers wouldn't use the same methodology for the calculation of road earthworks.

Hypothesis: Volume surface method is a valid and accurate method to calculate the earthworks for road embankments and excavations.

2 Equations for earthwork calculation

According to Autodesk Civil 3D online help manual [1] for earthworks calculation you can use average end area equation or Prismoidal equation.

2.1 Average end area equation

This method approximates the volume of a soil mass by calculating the average area of two end sections and multiplying it by the distance between them. The advantage of this method is that it is quite simple to use and it can be done manually without using complex software. Average end area equation works well with minimal variations in cross section such as trapezoidal excavations.

$$V_{EA} = L \cdot \frac{(A_1 + A_2)}{2} \tag{1}$$

The downside of the equation is that it assumes a uniform cross-section between the ends, which might not be realistic for uneven terrain or complex shapes. This can lead to overestimation or underestimation of the actual volume, depending on the specific case.
Despite its unreliable approach to calculation the earthworks, it is still widely used everywhere in the world. This equation is suitable for calculating rough estimates which are important for initial planning stages.

2.2 Prismoidal equations

The prismoidal equations are more advanced compared to the average end-area equations for calculating the volume of earthworks in road construction. They provide a more accurate estimate, especially for situations with irregular terrain or non-uniform road cross sections. A prismoid is a solid shape with parallel flat ends (bases) that are not necessarily identical or have the same number of sides. The sides connecting the bases are flat and extend throughout the length of the solid. Many road cuttings or embankments can be approximated as prismoids.
Prismoidal formulas often also add a prismoidal correction for highly irregular shapes. The prismoidal correction is a crucial concept to consider when using the prismoidal formula for volume calculations. It addresses the inherent limitation of the formula, which assumes a straight-sided prismoid with parallel faces. Many earthwork shapes encountered in civil engineering, such as uneven terrain or non-uniform road cross sections, might have bulging or tapering sides. The classic Prismoidal equation is given below.

$$V_{P0} = L \cdot \frac{(A_1 + 4A_m + A_2)}{6} \tag{2}$$

2.2.1 Mid-section area equation

Since this prismoidal equation Eq. 2. can be derived into Eq. 1. as shown by Deakin and Hunter [2], the prismoidal mid section area equation will be used. This equation calculates more accurate volume estimation compared to the average end area equation Eq. 1. and it also accounts for variations in the road cross section between the two ends. One caveat is that it requires additional calculations to determine the mid-section area (A_3). It is slightly more complex to apply compared to end area method.

$$V_{P1} = L \cdot \frac{(A_1 + A_3 + A_2)}{3} \tag{3}$$

2.2.2 Prismoidal mid-section approximation equation

This prismoidal mid-section approximation equation calculates the volume between two sectional areas, but it also adds the approximation of the area of the square root of their product which balances the results [2].

$$V_{P2} = L \cdot \frac{A_1 + A_2 + \sqrt[2]{A_1 \cdot A_2}}{3} \tag{4}$$

L – is perpendicular distance between the areas A_1 and A_2
A_1 – is the area at the start of the cross section
A_2 – is the area at the end of the cross section
A_3 – is the area in the mid section between A_1 and A_2
A_m – is sum of areas A_1 and A_2 divided by 2

$$A_m = \frac{A_1 + A_2}{2} \tag{5}$$

3 Volume surface method

Modern civil engineering software offer powerful tools for streamlining earthwork calculations and circumventing existing methods for volume calculation. In Civil 3D online help file [3] the volume surface is portrayed as a difference between a base surface and comparison surface which means that Z-value (vertical value) of any given point is the difference between Z-values of the surfaces. In the infrastructure context, volume surface calculates the difference between new, designed top surface or designed planum surface and existing ground which provide accurate calculation results. The volume surface method could become a cornerstone method in civil engineering for calculating earthwork volumes. This method leverages digital terrain models (DTMs) to estimate the amount of soil cut and fill required during construction projects.Advantages of using volume surface method range from efficiency and automation, improved surface analysis to correct volume calculation.

Figure 1 Existing ground surface and designed top surface

Volume surface method needs several surfaces to perform correct calculation:
• Existing ground surface (EG)
• Design Top surface (DTS)
• Design Planum surface (DPS)

3.1 Embankment calculation

In order to calculate the embankment, the designer needs to have existing ground surface (EG) obtained from the surveyor and designed road surface that includes shoulders, slopes, and curbs and so on. The designer then proceeds to calculate the volume between EG and DTS. The calculation will provide both positive (fill) and negative summary (cut) of earthworks. To get accurate results only the positive value is taken for further analysis. The positive volume (fill volume) is then subtracted by the volume of pavement construction. Negative result (cut volume) is not taken further into analysis because it does not consider planum surface in cut sections of the road.

3.2 Excavation calculation

When calculating the excavation volume using the volume surface method, the road designer compares EG with DPS (Design Planum surface). Modelling the DPS is arguably a lot harder than modelling the top surface of the road. The calculation will provide negative summary (cut) for the earthworks and positive summary for the fill volume. As explained in previous paragraph, only the negative value is applied for the result section.

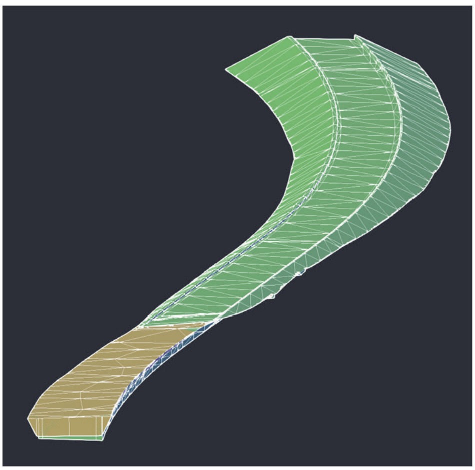

Figure 2 3D solid element of cut and fill

3.3 Challenges of volume surface method

Volume surface method can be very challenging to calculate. One road can have many different cross sections depending on the natural topology, geology and applied technical solutions. During the design of road that contains many different cross sections, it would be appropriate to divide the road into smaller sections and perform the calculation afterwards.

4 Test

The goal of the test is to calculate the embankment and excavation on 4 different road sections using various equations and comparing the results.
Autodesk Civil 3D and Excel software will be used to calculate the earthworks. The test will be performed on 4 different road sections with approximate length of 1 km. All alignments and profiles are designed according to Croatian legislative. Examples of the used cross section can be found at Fig. 3. or Fig 4. for expressway and Fig. 5. for state road. The test will produce result using the calculation of Eq. 1., Eq. 3., volume surface method and 3D solid (BIM) method.

Table 1 Layout of road and topology parameters for the testing

Topology	Cross section for the national road $V_p = 50$ km/h	Cross section for the expressway $V_p = 80$ km/h
Lowland topology	Test R1	Test R4
Hilly topology	Test R2	Test R3

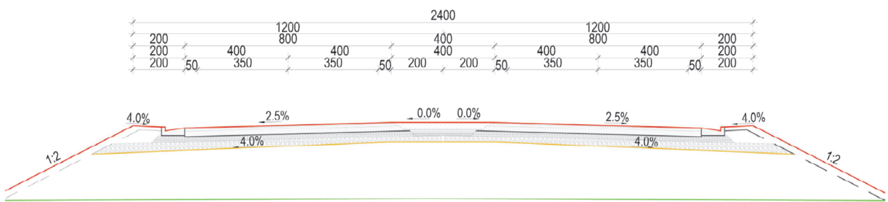

Figure 3 Normal cross section for expressway in embankment scenario

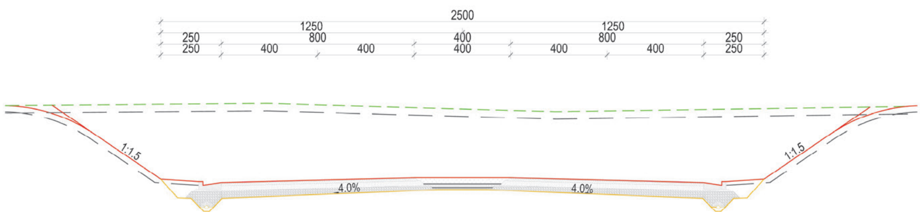

Figure 4 Normal cross section for expressway in excavation scenario

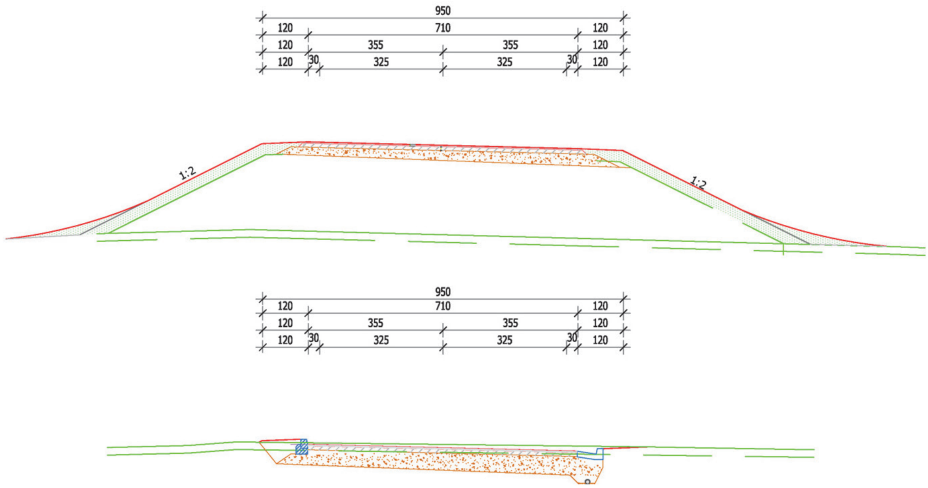

Figure 5 Normal cross sections for state road in both embankment and excavation scenarios

5 Results

For the purpose of this paper, only a summary of the results is displayed.

Table 2 Comparison of the results for the volume of excavation works

Excavation works	**Average end area Eq. 1** [m³]	**Prismoidal formula Eq. 3** [m³]	**Volume surface calculation** [m³]	**BIM 3D Model** [m³]
Test R1	2106,07	2028,17	2111,52	2111,52
Test R2	8502,76	8341,68	8582,60	8582,60
Test R3	17840,58	17804,58	17825,40	17825,40
Test R4	47759,10	47376,94	47746,18	47746,18

Table 3 Comparison of the results for the volume of embankment works

Excavation works	**Average end area Eq. 1** [m³]	**Prismoidal formula Eq. 3** [m³]	**Volume surface calculation** [m³]	**BIM 3D Model** [m³]
Test R1	27357,54	27303,43	27274,25	27274,25
Test R2	71956,52	71719,64	72067,30	72067,30
Test R3	84997,20	84939,3	86023,44	86023,44
Test R4	57360,37	57356,44	57761,48	57761,48

Table 4. and Table 5. show that volume surface method produces similar results with variations of ± 1 % for both cut and fill volumes in comparison to average end area equation which proves that it is an acceptable method for calculating infrastructure earthworks.

Table 4 Analysis of the results for excavation volume compared to Eq. 1.

	Average End Area Eq. 1 [m^3]	**Prismoid Formula Eq. 3 diff.** [%]	**Prismoid Formula Eq. 3 diff.** [m^3/per km]	**Volume surface diff.** [%]	**Volume surface diff.** [m^3/per km]
Test R1	2106,07	-3,70%	-77,90	0,26%	5,45
Test R2	8502,76	-1,89%	-161,08	0,94%	79,84
Test R3	17840,58	-0,20%	-36,00	-0,09%	-15,18
Test R4	47759,10	-0,80%	-382,16	-0,03%	-12,92

Table 5 Analysis of the results for embankment volume compared to Eq. 1.

	Average End Area Eq. 1 [m^3]	**Prismoid Formula Eq. 3 diff.** [%]	**Prismoid Formula Eq. 3 diff.** [m^3/per km]	**Volume surface diff.** [%]	**Volume surface diff.** [m^3/per km]
Test R1	27357,54	-0,20%	-54,11	-0,30%	-83,29
Test R2	71956,52	-0,33%	-236,88	0,15%	110,78
Test R3	84997,20	-0,07%	-57,90	1,21%	1026,24
Test R4	57360,37	-0,01%	-3,93	0,70%	401,11

6 Conclusion

In conclusion, there are clear benefits from using the volume surfaces method instead of other equations for calculating earthworks. The volume surface method offers a valuable tool for accurate and efficient volume calculations in infrastructure projects. Its ability to handle complex terrains and provide visual outputs makes it a powerful asset for engineers and construction professionals. The most enunciated benefit is the accuracy of the volume in comparison to the trapezoidal and prismoidal equations. The clear downside is that the road engineer needs to be able to model both the designed top surface and planum surface.

References

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