



## SPEED PROFILE CALCULATION WITH AN ACCELERATION MODEL: CASE STUDY OF ZAGREB TRAMWAY SYSTEM

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### Abstract

Knowledge of the speed profile of electric tramway vehicles is crucial for optimizing tramway traffic management procedures. This includes creating and optimizing timetables, calculating energy consumption, and maintaining rail infrastructure. The speed profile is influenced by many parameters, such as the maximum permitted speed and acceleration of the tram, the distance between stops, and the dwell time. In the past, a simplified model that assumed constant acceleration and deceleration was used to determine the speed profile of tramway vehicles. However, this model can lead to inaccuracies, especially for vehicles with high acceleration or deceleration capabilities. A more accurate approach is to use a model that considers the actual acceleration profile of the train. This can be done using data collected from GPS sensors to track the speed of the train over time. This paper presents a comparative evaluation of three acceleration models: a simple linear model, a linear multi-regime model, and a polynomial model, created from the data collected on the Zagreb tramway network. The GNSS (Global Navigation Satellite System) device was used to collect data on the acceleration and deceleration of the vehicle. Using proposed model to calculate the speed profile and timetable with an acceleration profile has several advantages. Firstly, it can improve the accuracy of the speed profile and timetable, which can lead to higher efficiency and reliability. Secondly, the timetable can be optimised for specific requirements, e.g. to minimise travel time or maximise passenger capacity.

*Keywords: traffic management, acceleration model, timetable*

### 1 Introduction

The management of the tramway system is a complex interdisciplinary process involving various aspects such as construction, operation, maintenance of infrastructure and rolling stock, and the organisation of traffic. Its main objective is to ensure a safe and comfortable ride while maximising the use of the tracks with minimum maintenance [1].

In today's rapidly evolving world, timely management of the railway system is crucial to maintaining competitiveness in the transport industry. Efficient systems attract passengers and ensure that rail transport remains relevant and competitive compared to other modes of transport [2].

The speed profile illustrates the change in vehicle speed over time, whereby the speed is generally reduced along certain track sections (at crossings, turnouts, and stops). The condition of the track also influences the speed profile, with speed being reduced on sections of the track that require maintenance or repair [3, 4].

Understanding the speed profile of tramway vehicles is one of the most important prerequisites for optimising tramway traffic management procedures. It enables the creation of accurate timetables, maximises tramway utilisation and facilitates better management of track maintenance.

The speed profile of the vehicle depends on the drivers' capabilities, type of vehicle, and traffic conditions [5]. Therefore, the calculation of the vehicle's speed profile is primarily based on factors such as the permitted speed on the observed section, the acceleration or deceleration profile of the vehicle, the presence of intersections, signalling systems and the distance between stops.

In the past, the speed profile was usually calculated using a simplified acceleration model that assumes constant acceleration. However, this model can lead to inaccuracies, especially for tramways with high acceleration or deceleration capabilities. More complex acceleration models describe the change in acceleration over time using linear-decreasing, polynomial or multi-term sinusoidal functions. According to research [5], polynomial and multi-term sinusoidal acceleration models provide more accurate predictions of the acceleration distance compared to constant and linear-decreasing acceleration models.

The rapid development of information technology, and measuring devices, has changed the nature and scope of research. The utilization of ubiquitous GNSS devices installed in public transport vehicles enables real-time vehicle tracking during passenger transport service. Systematic tracking of vehicles provides opportunities for numerous analyses, all aimed at improving system management.

This research investigates the performance of three proposed acceleration models for tramway vehicles: a simple linear acceleration model that assumes constant acceleration during the acceleration time; a linear multi-stage acceleration model that assumes that the vehicle acceleration occurs in three different stages, each defined by a simple linear function; and a polynomial acceleration model that describes the acceleration process with a polynomial function. The research is based on big data describing the change in the speed of tramway vehicles during passenger transport. The data was collected with a GNSS measurement device attached to a Zagreb TMK 2200 tram over seven operating days. Various queries were used to isolate data describing the typical acceleration process of tram vehicles as a function of the vehicle's final speed.

The accuracy of the proposed models was evaluated by comparing the estimated vehicle speed and distance travelled with the observed values. Although both the linear multi-stage and polynomial acceleration models provide similar results, the application of the linear multi-stage model provides slightly more flexibility in fitting the acceleration function to the observed data.

## 2 Data collection and database creation

The acceleration of the tramway is influenced by a combination of vehicle and traffic characteristics. The vehicle characteristics include the power of the traction motor, the mass and load of the tramway and the maximum acceleration of the vehicle. Traffic characteristics include whether the tramway runs in separate corridor or shares the surface with other vehicles, the distance between stops and the maximum speed the tramway can reach on a particular section.

To understand how these factors influence acceleration in real-life operations, big data describing the change in tramway speed over time was collected. The data was collected using a Sensornet SAM Infra measurement device mounted on a TMK 2200 tram for long-term monitoring of tram track [6]. This device accurately recorded the speed and position changes of the vehicle over time in 1s interval. A sample of seven operating days was analysed on a 25 km long section of tramway line.

The collected data was filtered and structured in a database. The data processing performed in SQL Server aimed to identify and group data describing individual tramway accelerations and classify them according to the final speed. The acceleration profiles were then calculated based on the observed speed profiles as the median speed of all vehicles with the same final speed. A total of 3,982 vehicle acceleration events were identified, including 491 accelerations with a final speed of 30 km/h, 584 accelerations with a final speed of 40 km/h and 674 accelerations with a final speed of 50 km/h.

The acceleration and speed profiles as function of final speed are presented in Figure 1. In the composite view, the final speed is classified by colour, from light to dark blue, the values of the acceleration profile are marked with squares, and the values of the speed profile with circles.

The shape of the speed profile can be described as an S-line, with shallow concave at the start of acceleration and deep convex until the final speed is reached. Converted into an acceleration profile, two acceleration stages can be distinguished: a sudden increase in the acceleration value until the highest value is reached and a decrease in the acceleration value until the final speed is reached.

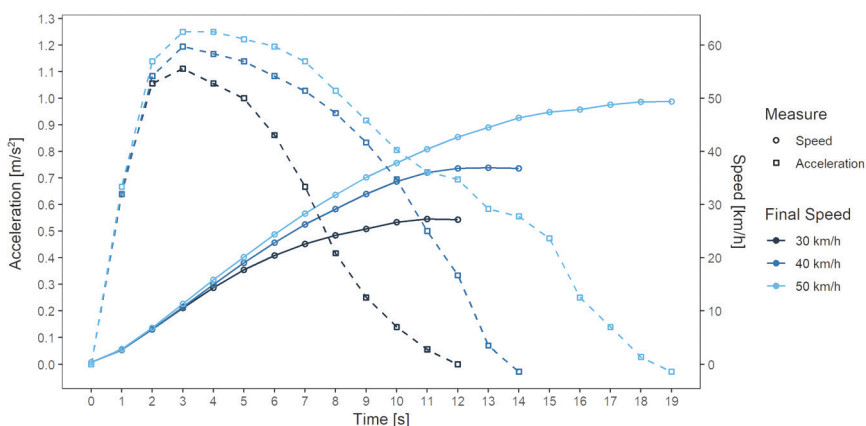


Figure 1 The acceleration and speed profiles according to the final speed

### 3 Modelling

The research involved the modelling of three types of acceleration functions: a simple linear, a linear multi-stage, and a polynomial acceleration model. Furthermore, the modelling was performed based on the final speed classes, 30, 40 and 50 km/h. A simple linear acceleration model assumes that the acceleration is constant during acceleration time, from the start  $t_0$  until the vehicle reaches its final speed  $v_f$ . The constant acceleration function (1) results in a linear increase in vehicle speed over time.

$$a(t) = \text{const. for } t \in [t_0, t_m] \tag{1}$$

A linear multi-stage acceleration model assumes that the acceleration of the vehicle occurs in three separate phases, each defined by a simple linear function. The first phase is described by a linearly increasing function within a time interval  $[t_0, t_1]$  (2), from the start of acceleration until the maximum acceleration value is reached,  $a_m$ . The second phase is described by a linear constant of the maximum acceleration value  $a_m$  (3) within a time interval

$[t_1, t_2]$ , while the third phase is described by a linearly decreasing function over time within a time interval  $[t_2, t_3]$  (4) until the final velocity  $v_f$  is reached.

$$a(t) = \frac{a_m}{t_1} \cdot t \text{ for } t \in [t_0, t_1] \quad (2)$$

$$a(t) = a_m \text{ for } t \in [t_1, t_2] \quad (3)$$

$$a(t) = a_m + \frac{a_m}{t_3 - t_2} (t - t_2) \text{ for } t \in [t_2, t_3] \quad (4)$$

where:

$a_m$  – maximum acceleration,

$t_1$  – time at the end of the first acceleration phase,

$t_2$  – time at the end of the second acceleration phase,

$t_3$  – time at the end of the third acceleration phase.

A polynomial acceleration model [5] assumes that the acceleration of the vehicle changes over time, from the beginning of the acceleration  $t_0$  until the final speed  $v_f$  is reached, according to a polynomial function determined by equation (5).

$$a(t) = \frac{\left[ (1 + 2m)^{2+1/m} \right]}{4m^2} \cdot a_m \theta^n (1 - \theta^m)^2 \quad (5)$$

where:

$a_m$  – maximum acceleration,

$\theta$  – time ratio  $t/t_a$ ,

$t_m$  – time to achieve maximum acceleration,

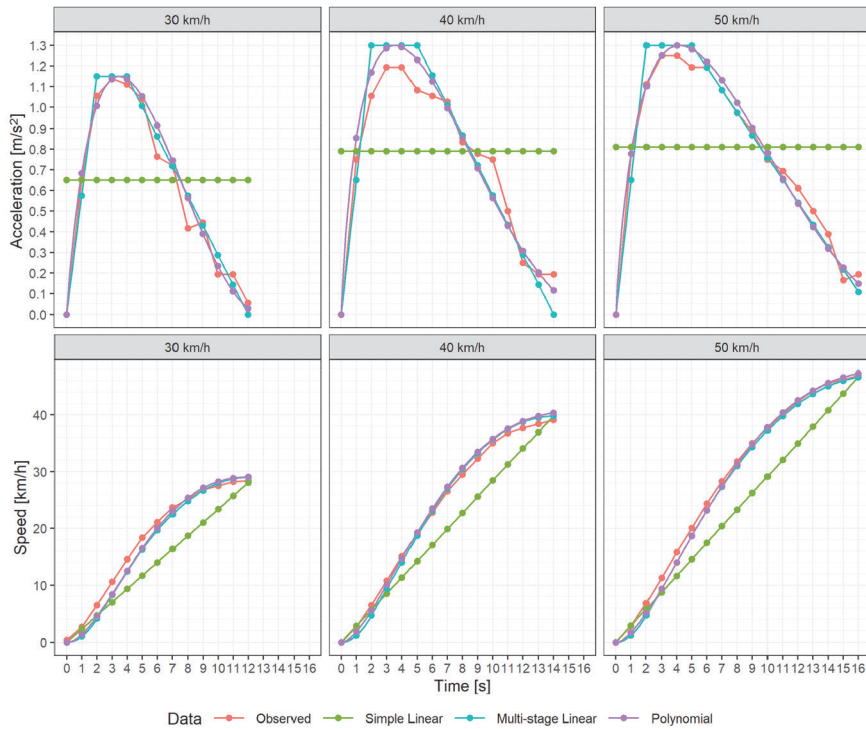
$t_a$  – acceleration time,

$m, n$  – parameters to be determined.

Based on the observed values of speed or acceleration changes over time for the corresponding final speed, the necessary parameters for the proposed acceleration models are determined. The parameters defining the proposed acceleration models are listed in Table 1. The results of the proposed acceleration models in the form of acceleration-time and speed-time diagrams are shown in Figure 2.

**Table 1** The parameters defining proposed acceleration models

Model type	Parameter	30 km/h	40 km/h	50 km/h
Simple Linear	$a_m$ [m/s <sup>2</sup> ]	0,65	0,79	0,81
	$a_m$ [m/s <sup>2</sup> ]	1,15	1,30	1,30
Multi-stage Linear	$t_1$ [s]	2,0	2,0	2,0
	$t_2$ [s]	4,0	5,0	5,0
	$t_3$ [s]	12,0	14,0	17,0
Polynomial	$a_m$ [m/s <sup>2</sup> ]	1,15	1,30	1,30
	$t_m$ [s]	3,0	3,0	3,0
	$t_0$ [s]	13,0	17,0	20,0
	$m$ [-]	0,6	0,3	0,3
	$n$ [-]	1,0	1,0	1,0



**Figure 2** Speed-time profile

## 4 Results

The quality of the proposed acceleration models was evaluated by comparing the observed and estimated tramway speed over the acceleration time. The proposed acceleration models accurately predicted the final speed, as shown in Figure 3. However, the simple linear model significantly underestimated the speed throughout the acceleration process, with deviations of up to 10 km/h. In contrast, the multi-stage and polynomial models closely matched the observed speeds, with deviations typically within 2 km/h (both positive and negative).

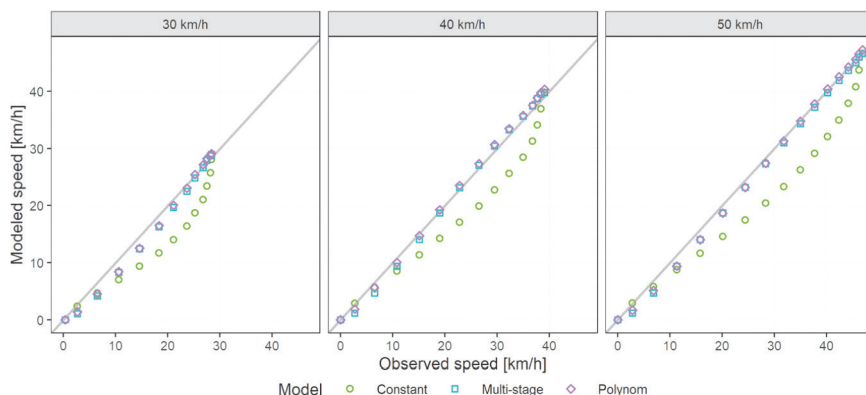


Figure 3 Modelled vs observed speed

When comparing the distances predicted by the models, the simple linear model also showed the greatest deviations from the real data. The average distance errors increased with increasing final speed, ranging from -6.5 to -10.1 metres with a standard deviation of 5.5 to 8.8 metres. In contrast, the multi-stage and polynomial models had much smaller and more consistent errors. Their average errors ranged from -2.9 to -0.9 metres and -3.0 to -0.2 metres, respectively, with a standard deviation of around 1 metre for both (as shown in Table 2).

Table 2 Mean and SD of Error in distance estimation

Final speed	Error [m]	Simple Linear	Multi-stage Linear	Polynomial
50 km/h	Mean	-10.1	-2.9	-0.2
	SD	8.8	1.4	1.0
40 km/h	Mean	-6.6	-0.9	-0.9
	SD	5.9	0.7	0.8
30 km/h	Mean	-6.5	-2.6	-3.0
	SD	5.5	1.4	1.1

Overall, both the multi-stage and polynomial models effectively capture the relationship between tram acceleration and final speed. Their minimum deviations in speed and distance estimates are within an acceptable range. However, due to its inherent structure, the multi-stage model offers slightly more flexibility in fitting the acceleration function to the observed data.

## 5 Conclusion

The use of acceleration models for the calculation of speed profiles and timetables offers numerous advantages. Firstly, it can enhance the accuracy of speed profiles and timetables, leading to greater efficiency and reliability. Secondly, timetables can be optimised to meet specific requirements, such as reducing journey time or increasing passenger capacity.

The speed profile of a vehicle varies from driver to driver and depends on the type of vehicle and traffic conditions. By applying big data describing vehicle tracking during real-time operation, patterns in tram acceleration can be identified based on final speed.

Complex acceleration models, such as the linear multi-stage or polynomial models, allow accurate estimation of speed and distance travelled based on final speed, which facilitates the creation of accurate speed profiles and timetables. The estimation of distance travelled by both proposed models is within acceptable limits, but due to its simplicity and slightly greater flexibility in fitting the acceleration function to the observed data, the application of the linear multi-stage acceleration model is recommended. This model allows a more straightforward implementation and still provides sufficient accuracy for practical applications.

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