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THE INFLUENCE OF A THIN SOIL LAYER ON THE RAYLEIGH WAVE DISPERSION CURVE

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Abstract

During the construction of road and railway infrastructure, there is a continuous need for the foundation of heavy structures and embankments on soft soils, or soils of inadequate bearing capacity. In such cases, some of the soil improvement methods such as stone columns or jet grouting are applied. An average increase of stiffness for the improved soil can successfully and reliably be determined by one of the non-destructive surface wave methods. The most common are: Spectral Analyses of Surface Waves, Continuous Surface Wave analysis, and Multichannel Analysis of Surface Waves. The methods are based on dispersive characteristics of Rayleigh waves, which consider surface waves of different wavelengths, i.e., frequencies, spread to different depths. Thus, waves of lower frequency, meaning of greater wavelength, are spreading deeper in the medium than waves of high frequency and smaller wavelength. The change of wave-spreading velocity on the surface depending on the wavelength is called wave dispersion and is closely related to the stiffness characteristics of a multi-layered medium through which the wave is spreading. The paper presents a detailed parametric analysis performed to identify the influence of a thin weak and stiff soil layer on the Rayleigh wave dispersion curve.

Keywords: Rayleigh wave, dispersion curve, stiffness, soil improvement

1 Introduction

In the process of road and railway infrastructure construction, a persistent demand arises for establishing foundations for robust structures and embankments on soft soils or soils with insufficient bearing capacity. In such scenarios, soil improvement techniques, such as the implementation of stone columns or jet grouting, are employed. By employing soil improvement methods, it is possible to enhance its bearing capacity, reduce or control both overall and differential settlements, decrease the time required for deformations to occur, lower soil permeability, eliminate water from the soil by establishing internal drainage systems, and enhance erosion stability. By achieving the desired degree of soil improvement, coupled with the selection of appropriate foundation construction and performance monitoring methods, it becomes feasible to quantify the success of implemented interventions and manage associated risks [1].

During the ground improvement processes, considerable emphasis was placed on quality control for the newly formed materials. The complex state of stress and the interaction between the gravel columns and the surrounding soil make it impossible to determine the stiffness of the composite by local testing of the components. To determine the degree of soil improvement, it is necessary to conduct a test that will cover a larger volume of the improved soil and determine its average newly formed stiffness characteristics.

The application of non-destructive surface wave methods enables the successful and reliable determination of the average stiffness increase in the improved soil [2]. The most common are: Spectral Analysis of Surface Waves (SASW), Continuous Surface Wave (CSW) analysis, and Multichannel Analysis of Surface Waves (MASW).
Non-destructive methods are based on the dispersive characteristics of Rayleigh waves, considering the fact that surface Rayleigh waves of different wavelengths, or frequencies, spread to different depths. Thus, waves of lower frequency, and therefore longer wavelength, propagate deeper into the medium than waves of high frequency, i.e., shorter wavelength. In a layered medium, the velocity of wave propagation on the surface depends on the frequency, that is, the wavelength of the wave [3]. This change in the velocity of wave propagation on a surface with wavelength is called wave dispersion and is closely related to the stiffness characteristics of the layered medium through which the wave passes. The dispersion curve represents the dependence of the wave velocity on the surface and the wavelength or frequency.

2 Theoretical dispersion curves of Rayleigh waves in horizontally layered soil

The theoretical dispersion curve represents the solution of plane Rayleigh wave propagation based on the Haskell-Thomson matrix formulation of wave propagation in layered media [4, 5]. In this formulation, the dispersion curve represents the eigenvalues of the transfer matrix of the layered medium. Kausel and Roësset [6] presented an alternative determination of the theoretical dispersion curve by formulating a dynamic layered soil stiffness matrix similar to that used in the finite element method. For analytical and computational modeling, each layer of the soil model is assumed to be horizontal, homogeneous, isotropic and elastic. A Rayleigh wave is viewed as a plane wave propagating in a 2D medium.
In the dynamic stiffness matrix, layer properties [layer thickness h (in m), density ρ (in kg/m^3), Poisson's ratio v (-), shear wave velocity V_S (in m/s) and longitudinal wave velocity V_P (in m/s)] (Figure 1) and wave properties [frequency f (in Hz) and wave velocity V (in m/s)] are represented by 2×2 submatrices K_{ij}^k which form the layer stiffness matrix K^k for the **k**-th layer. The layer stiffness matrices are then assembled into a larger matrix called the global stiffness matrix K_{glob}. The global stiffness matrix is square and symmetric, with matrix dimension $2n + 2$, where n is the number of layers in the elastic half-space.

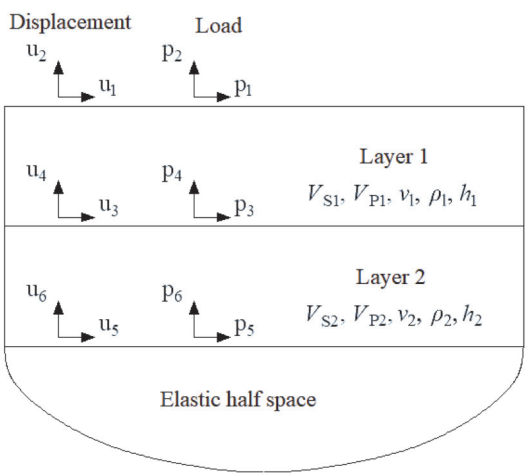

Figure 1 Layered media with two layers above an elastic half-space

The global stiffness matrix gives the relation of the displacement vector u to the load vector p on the layer interface as follows:

$$K_{glob}u = p \tag{1}$$

For a layered medium with two layers located above an elastic half-space, as shown in Figure 1 (ie $n = 2$), the load vector (p) and the displacement vector (u) consist of $2n+2$ elements:

$$p = [p_1 p_2 p_3 p_4 p_5 p_6]^T \tag{2}$$

$$u = [u_1 u_2 u_3 u_4 u_5 u_6]^T \tag{2}$$

The stiffness matrices for the first and second layer are:

$$K^1 = 2kG_1\begin{bmatrix} K_{11}{}^1 & K_{12}{}^1 \\ K_{21}{}^1 & K_{22}{}^1 \end{bmatrix} \tag{4}$$

$$K^2 = 2kG_2\begin{bmatrix} K_{11}{}^2 & K_{12}{}^2 \\ K_{21}{}^2 & K_{22}{}^2 \end{bmatrix} \tag{5}$$

where the submatrices of the ith layer are defined as:

$$K_{11}{}^i = \frac{1 - s^2}{2D}\cdot\begin{bmatrix} \frac{1}{s}\left(C^r S^s - rsC^s S^r\right) & -\left(1 - C^r C^s - rsS^r S^s\right) \\ -\left(1 - C^r C^s + rsS^r S^s\right) & \frac{1}{r}\left(C^s S^r - rsC^r S^s\right) \end{bmatrix} - \frac{1 + s^2}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{6}$$

$$K_{12}{}^i = \frac{1 - s^2}{2D}\cdot\begin{bmatrix} \frac{1}{s}\left(rsS^r - S^s\right) & -\left(C^r - C^s\right) \\ \left(C^r - C^s\right) & \frac{1}{r}\left(rsS^s - S^r\right) \end{bmatrix} \tag{7}$$

$$K_{21}{}^i = \frac{1 - s^2}{2D}\cdot\begin{bmatrix} \frac{1}{s}\left(rsS^r - S^s\right) & -\left(C^r - C^s\right) \\ \left(C^r - C^s\right) & \frac{1}{r}\left(rsS^s - S^r\right) \end{bmatrix} \tag{8}$$

$$K_{22}{}^i = \frac{1 - s^2}{2D}\cdot\begin{bmatrix} \frac{1}{s}\left(C^r S^s - rsC^s S^r\right) & \left(1 - C^r C^s + rsS^r S^s\right) \\ \left(1 - C^r C^s + rsS^r S^s\right) & \frac{1}{r}\left(C^s S^r - rsC^r S^s\right) \end{bmatrix} - \frac{1+s^2}{2}\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \tag{9}$$

The half-space stiffness matrix is defined as:

$$K_H = 2kG\left(\frac{(1 - s^2)}{2(1 - rs)}\begin{bmatrix} r & 1 \\ 1 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\right) \tag{10}$$

The global stiffness matrix for the layered medium, i.e. the soil profile, is obtained by joining the submatrices for each layer and for the elastic half-space:

$$K_{glob} = \begin{bmatrix} K_{11}^{\ 1} & K_{12}^{\ 1} & \\ K_{21}^{\ 1} & K_{22}^{\ 1} + K_{11}^{\ 2} & K_{12}^{\ 2} \\ & K_{21}^{\ 2} & K_{22}^{\ 2} + K_H \end{bmatrix} \quad (11)$$

The parameters in expressions (4) – (10) are defined as follows:

$$k = \frac{2\pi}{L} = \frac{2\pi f}{V_R} (k > 0) \quad (12)$$

$$D = 2\left(1 - C^r C^s\right) + \left(\frac{1}{rs} + rs\right) S^r S^s \quad (13)$$

$$r = \sqrt{1 - \left(\frac{\omega}{k V_P}\right)^2} \quad (14)$$

$$s = \sqrt{1 - \left(\frac{\omega}{k V_S}\right)^2} \quad (15)$$

$$\omega = 2\pi f \ (\omega > 0) \quad (16)$$

$$C^r = \cosh(krh);\ C^s = \cosh(ksh);\ S^r = \sinh(krh):\ S^s = \sinh(ksh) \quad (17)$$

where:
ω – angular frequency,
f – frequency,
L – wavelength,
V – wave velocity
h – layer thickness.

The Rayleigh wave dispersion curve mode can be characterized as a natural mode of wave propagation where there is a displacement (ie $u \neq 0$) when there is no load (ie $p = 0$) and when there is no incoming wave in the elastic half-space [7]. In other words, the Rayleigh wave mode satisfies the following equation:

$$K_{glob} u = 0, \text{ where } u \neq 0 \quad (18)$$

To obtain a non-trivial solution for u, K_{glob} must be singular. The singularity of a matrix is theoretically evaluated by the determinant of the matrix. If the determinant is zero, the matrix is singular.
For the purpose of determining the dynamic stiffness matrix, a Python program code was developed for the calculation of the first three modes of the dispersion curve.

3 The influence of a thin layer on the dispersion curves of Rayleigh waves

A program code developed in the Python programming language was used to analyze the influence of the thin layer on the theoretical dispersion curves. The investigation of the influence of the thin layer was carried out by considering three modes of Rayleigh waves (fundamental mode, first mode, and second mode).
A soil profile with a total thickness of 30 m was used for the analysis, whereby the influence of a thin layer located at three different depths in the soil was considered. The analysis was performed with a thin layer at the top of the profile, in the middle, and at the bottom of the profile, and its thickness varied between 0.1 m (0.33% of the total thickness of the analyzed soil profile) and 1 m (3.33% of the total thickness of the analyzed profile soil), with a step of 0.1 m. For each such soil profile, dispersion curves were calculated for a layer that has a much higher stiffness and a layer that has a much lower stiffness than the rest of the soil.
Since the stiffness is calculated as the product of the soil density ρ and the square of the shear wave velocity VS, the variation of the stiffness is obtained by specifying the corresponding shear wave velocity. To obtain a much higher stiffness of the thin layer, the value of the shear wave velocity was chosen to be twice as high as the rest of the profile, while to obtain a much lower stiffness, the velocity was chosen to be twice as low as the rest of the profile.
When calculating the dispersion curves, a range was selected in which the wave frequency and shear wave velocity will vary. All analyses were performed for the same selected ranges. The obtained dispersion curves were compared with the dispersion curves obtained for a homogeneous soil layer 30 m thick, with constant stiffness per depth. Based on these reference dispersion curves, the deviation of the dispersion curves obtained for the soil profile containing a thin layer of higher or lower stiffness at a certain depth was calculated.
Figure 2 shows an example of calculating the deviation of the dispersion curves. For a thin layer thickness of 0.1 m at the top of the profile, deviations in all three modes of Rayleigh waves were obtained. A deviation of 1.72% was obtained in the fundamental mode, 0.58% in the first mode, and 0.49% in the second mode. The cumulative deviation is 2.79%. With this stiffness distribution, for the thin layer on top, the deviation in the fundamental mode is significantly higher than the deviation in the first and second modes, with the deviation in the second mode being smaller than the deviation in the first mode.
Figure 3 shows the relationship between the thickness of the thin layer and the deviations of the dispersion curves of all profiles from the dispersion curves of the reference profile for the thin layer located on top of the soil profile and which is much stiffer.
Increasing the thickness of the thin layer increases the deviation from the dispersion curves for the reference soil model. The deviation is largest in the fundamental mode, smaller in the first mode, and smallest in the second mode.
In this case, even with a thickness of a thin layer of 0.5 m, the deviation only in the fundamental mode exceeds 4% (cumulatively almost 9%). For a layer with a thickness of 1 m, the total deviation is significantly higher, i.e., 5.92% for the fundamental mode, 4.96% for the first mode, and 3.03% for the second mode cumulatively giving a deviation of 13.91%.

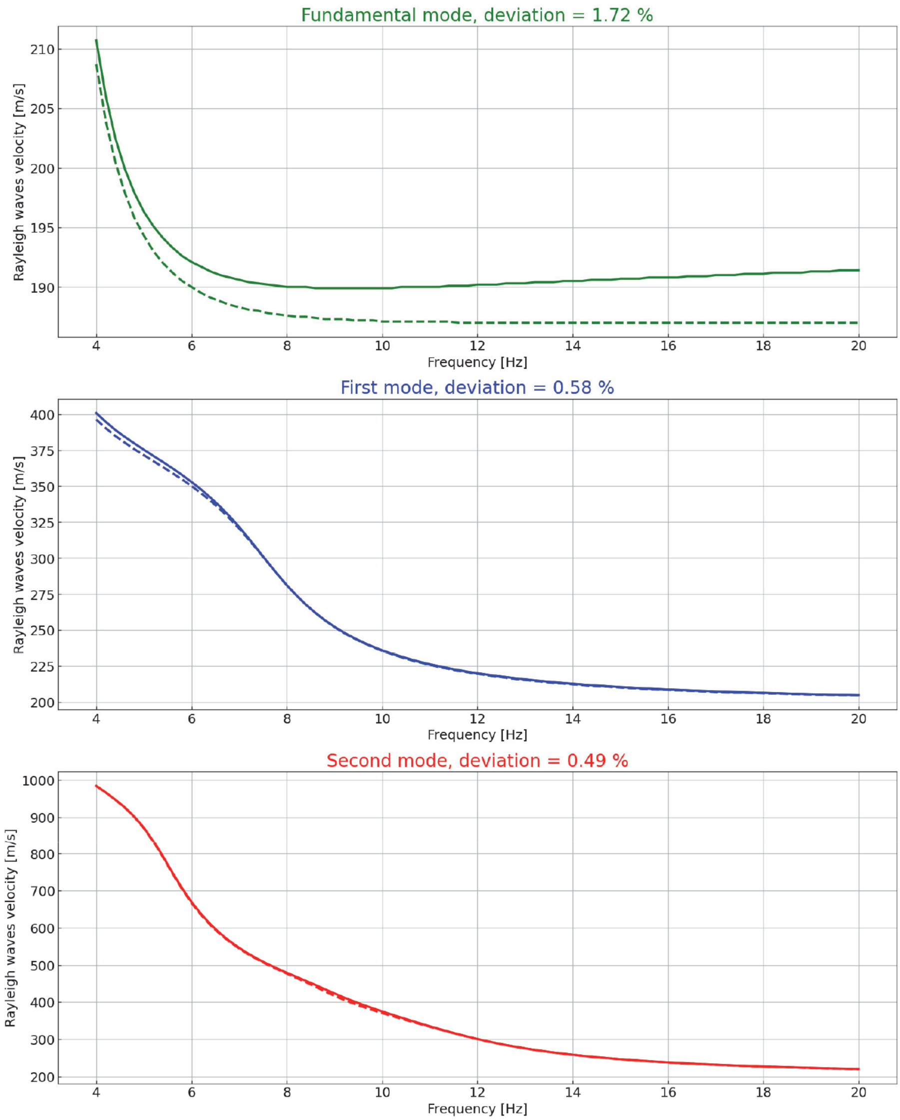

Figure 2 A thin layer of higher stiffness on top of the profile - thickness 0.1 m

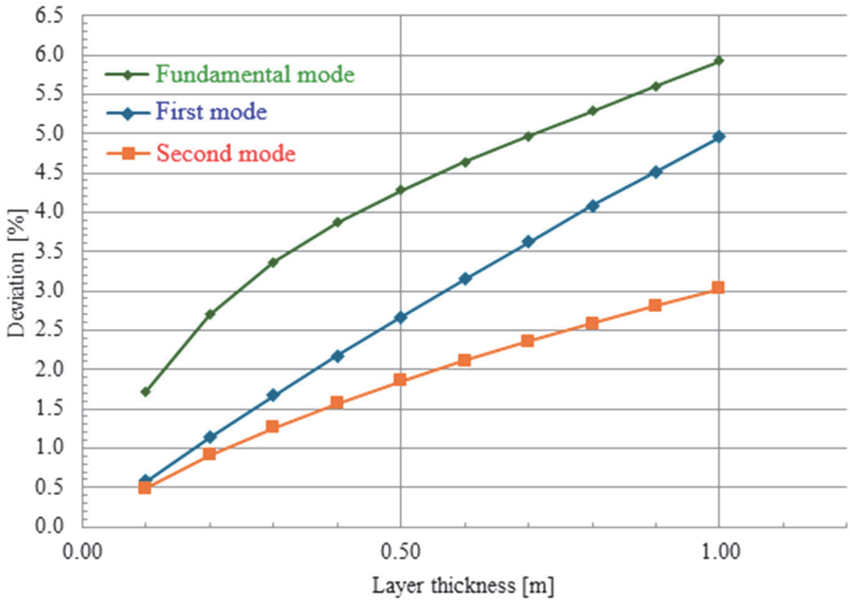

Figure 3 Deviations of the results for a thin layer of higher stiffness on top of the profile - thickness 0.1 m

4 Conclusion

The velocity of wave propagation on the surface depends on the frequency, i.e., the wavelength of the wave and is closely related to the stiffness characteristics of the soil through which the wave passes. The dynamic stiffness matrix is used to calculate the dispersion curves, i.e., to determine the relationship between the velocity of the Rayleigh wave and the properties of the material of the horizontally layered elastic medium.

Analyses of the influence of the thin layer located at the top of the soil profile, in the middle, and at the bottom of the profile were carried out. All analyses were performed for a thin layer that has a much higher stiffness compared to the surrounding soil and a much lower stiffness compared to the surrounding soil, and the obtained dispersion curves were compared with those obtained for a homogeneous 30 m thick soil layer, with constant stiffness per depth.

It is recommended that when determining the theoretical dispersion curve of Rayleigh waves in horizontally layered soil, a layer thickness of less than 1% should not be taken from the total analyzed thickness of the soil profile.

In relation to the reference dispersion curves, the deviation in all cases increases with the increase in the thickness of the thin layer. For the case of a thin layer with lower stiffness, the largest deviation in the fundamental mode was observed at the top layer, and the largest deviation in the first and second modes at the bottom layer. With a thin layer of higher stiffness, the largest deviation in the fundamental and first modes is for the layer on top, and the largest deviation in the second mode is for the layer in the middle.

If the cumulative deviation in the fundamental, first, and second modes is observed, for a thin layer of lower stiffness, the largest deviation is for the thin layer at the bottom, and the smallest for the thin layer at the top. For a thin layer of higher stiffness, the largest deviation was obtained for the layer at the top, and the smallest deviation for the thin layer at the bottom, which are also the extremes of deviation for all observed cases.

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